NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 6
COURSE CODE: LIA601S	**COURSE NAME:** LINEAR ALGEBRA 2
SESSION: JULY 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr S.N. NEOSSI NGUETCHUE AND Pr A. KAMUPINGENE
MODERATOR:	Mr B. OBABUEKI

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations. All numerical results must be given using 5 decimals where necessary unless mentioned otherwise.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Including this front page)

Attachments
None

QUESTION 1 [13 Marks]

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator defined by

$$T(x, y, z) = (2x + 3y, 4x - 5y, x + z).$$

Find the matrix representation of T relative to the basis
$e = \{v_1, v_2, v_3\} = \{(1, 0, 1), (0, 3, 0), (0, 0, 2)\}$.

QUESTION 2 [23 Marks]

Define the linear transformation $T: \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ by

$$T(ax^2 + bx + c) = ax^2 + (a + b)x + (a + b + c)$$

2.1. Determine whether $p(x) = x^2 + 2x + 3$ is in the range of T. [10]

2.2. Find a basis for the range of T. [5]

2.3. Find a basis for the kernel of T. [5]

2.4. Verify that the Rank Theorem holds. [3]

QUESTION 3 [50 Marks]

Let $A = \begin{pmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{pmatrix}$.

3.1. Find the minimal polynomial of A. [13]

3.2. Explain why A is diagonalizable or not diagonalizable. Give the full details of each statement made. [22]

3.3. Find a Jordan canonical form J of A. [5]

3.4. Find a matrix Q such that $Q^{-1}AQ = J$. [10]

QUESTION 4 [14 Marks]

Let A be a square matrix with minimal polynomial

$$m(t) = t^n + a_{n-1}t^{n-1} + \cdots + a_1 t + a_0.$$

4.1. Show that A is invertible if and only if $a_0 \neq 0$. [7]

4.2. Prove that if A is invertible, then [7]

$$A^{-1} = -\frac{1}{a_0}(A^{n-1} + a_{n-1}A^{n-2} + \cdots + a_1 I_n).$$

END OF PAPER
TOTAL MARKS: 100

God bless you !!!